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# PRIMARY MATHEMATICS

Knowledge and Understanding



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# 1

## Introduction

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# Introduction

## About this book

This book has been written to cater for the needs of primary trainees on all courses of initial teacher training in England and other parts of the UK. By the end of your course, you will be required to demonstrate your subject knowledge and understanding and your competence in using this knowledge in your teaching. A secure subject knowledge of mathematics is required for the award of Qualified Teacher Status (QTS) or its equivalent. This book will also be useful to newly qualified teachers (NQTs) and other professionals working in education who have identified aspects of their mathematics subject knowledge which require attention.

This book has been written with the Teachers' Standards firmly at its core. The Teachers' Standards in England (DfE, 2011c) came into force from 1 September 2012. They define the minimum level of practice expected of all teachers from the point of being awarded QTS.

There have been many changes in education in the last few years. Following the Tickell Report (Tickell, 2011), changes to the Early Years Foundation Stage (EYFS) were implemented, including a new Statutory Framework for the Early Years Foundation Stage, mandatory from September 2012 and updated in 2014. That led to the new National Curriculum in England, which maintained schools in England are legally required to follow. This book includes information on the programmes of study for National Curriculum mathematics and on the mathematics Early Learning Goals for children in the Early Years.

Features of the main chapters of this book include:

- clear links with the Teachers' Standards;
- information about the curriculum context, including the mathematics National Curriculum in England and the Statutory Framework for the Early Years Foundation Stage;
- mathematics knowledge and understanding;

- reflective and practical activities for you to undertake, many of which are related to pedagogy;
- research summaries that give additional background insights into how children's understanding of mathematical concepts develops;
- a summary of key learning points;
- suggestions for further reading on the aspect in question.

There are also self-assessment questions so that you can check on how well you have assimilated the knowledge and understanding. The answers to these questions are contained in a separate chapter. The book also contains a glossary of terms, and details of publications referenced in the main chapters.

## **A mathematics subject knowledge really does matter!**

A healthy subject knowledge of mathematics is widely acknowledged as a critical factor in the complex process of teaching mathematics itself (Ball and Bass, 2000; Morris, 2001; Rowland et al., 2001; TDA, 2007). Few nowadays would argue that planning, teaching and assessing mathematics lessons, setting learning outcomes, choosing appropriate activities and resources, identifying children's errors and misconceptions, asking and responding to questions, and so on, could be achieved without a sound knowledge of mathematics in the first place. Within primary schools currently there is a clear drive to raise standards in mathematics through raising expectations and national target setting.

Implicit within this drive to raise standards in mathematics is the requirement for teachers to have the necessary subject knowledge to teach confidently and effectively. Ma (1999) proposed that a *profound understanding of fundamental mathematics* (PUFM) is crucial to effective mathematics teaching. PUFM refers to the depth, breadth and thoroughness of the knowledge that is required to be an accomplished teacher of primary mathematics. Ma suggested that teachers with PUFM make connections between mathematical concepts and procedures from the simple to the complex, appreciate different facets of an idea and various approaches to a solution, are particularly aware of the simple but powerful foundational concepts and principles of mathematics, and are knowledgeable about the whole primary mathematics curriculum, not just the content of a particular age level. This may well seem like a large a burden. However, persisting with the

development of an appropriate level of mathematics subject knowledge will ensure confident mathematics teaching that motivates, challenges and extends the children.

## The importance of talking mathematics

As well as securing confidence in your subject knowledge of mathematics, you need to be ready to talk mathematics. Increasingly, teachers are seeing the benefits of talking about mathematics with children from a young age. This is a stepping stone to confident use of mathematical language later on. The following Research Summary outlines some of the theory behind this:

### RESEARCH SUMMARY

In the *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools*, Sir Peter Williams offered a number of recommendations for the long term to enhance the standing of the teaching profession and the mathematical learning of the children in their care (Williams, 2008, page 4). One of the issues raised was the need for high-quality talk in mathematics. Williams explains why the vitally important question of the classroom discussion of mathematics is an issue he singles out. He states, it is often suggested that *mathematics itself is a language* but it must not be overlooked that only by constructive dialogue in the medium of the English language in the classroom can logic and reasoning be fully developed - the factors at the very heart of embedded learning in mathematics (Williams, 2008, page 4).

Neil Mercer and his colleagues have undertaken research that tries to address the issues surrounding mathematical talk in primary classrooms and early years settings. They identified that:

- the teacher has a key role in enabling children to talk and reason together effectively;
- providing children with guidance and practice in how to use language for reasoning enables them to use it more effectively as a tool for working on mathematics problems together;
- improving the quality of children's use of language improves individuals' learning also;
- the teacher is an important model and guide for children's use of language and reasoning (Littleton et al., 2005; Mercer and Sams, 2006a, 2006b).

# The importance of reasoning for the development of your own mathematical knowledge

The development of your own mathematical reasoning skills has an impact not only on your own mathematical knowledge but also on the quality of your teaching.

## RESEARCH FOCUS

### Mathematical explanations

Charalambous et al. (2011) have written extensively about the development of teachers' mathematical explanations during the period of their teacher training. They followed a small group of pre-service teachers through their training, monitored the development of their mathematical explanations and explored the reasons for this development. The pre-service teachers' explanations at the beginning of their course were limited and tended to focus on how to carry out mathematical procedures, often reflecting the ways that they were taught themselves.

The teachers in the study progressed at very different rates in terms of their ability to give mathematical explanations. Those teachers who made the greatest gains in the quality of their explanations were those who, among other things, were able to develop their own ability to reason mathematically and to reconsider and refine their own ideas about what it means to reason mathematically. They said that being able to explain their ideas and thinking in a logical way was essential to improving their mathematical explanations.

## What do we mean by 'reasoning skills' and how do you develop them?

The Advisory Committee on Mathematics Education's (ACME) response to the draft National Curriculum called for a clear definition of mathematical reasoning and suggested the following:

## 1 Introduction

*Mathematical reasoning requires analysing information presented in different forms, recognising given information, identifying what additional information is needed and what forms of reasoning can provide it; identifying and conjecturing patterns, relationships, and generalisations; testing, inducing, deducing, and proving; and communicating ideas in mathematical language. (ACME, 2012, p.5)*

Although there is not one clear definition, it is clear that ‘reasoning’ skills include the following:

- The ability to make sense of mathematics, not just the ability to follow prescribed rules and procedures.
- The ability to know which mathematical procedure to apply to a problem, not just the ability to ‘do’ the procedure.
- The ability to estimate and discuss a likely result before completing any advanced calculations, not just the ability to find the answer.
- The ability to check your answers, via other methods, to ensure your calculations are correct, not just a need to be told that your answer is correct.

Marcus Witt (2014) offers an explanation of reasoning through an example of reasoning in the classroom:

### EXAMPLE

In Joyce’s room, the children are solving a puzzle which looks like the one below.

|    |    |    |    |    |
|----|----|----|----|----|
| ★  | ★  | ★  | ★  | 8  |
| £  | \$ | &  | ♣  | 17 |
| ★  | ♠  | ★  | ♠  | 16 |
| \$ | ★  | €  | ♣  | 11 |
| 9  | 11 | 14 | 18 |    |

The discussion centred on the ways in which the children had solved the puzzle:

Joyce: How did you start the puzzle?

Anish: I saw that the top row was all stars. There are four stars and together they come to 8, so I knew that the star must be worth 2.

Joyce: (Feigning not to understand fully) So, explain to me exactly how you knew each one was worth 2.

Anish: Well, there are four stars. The whole row comes to 8, so I knew that four stars equals 8, so each star must be worth 2. See 2 and 2 more is 4 and 2 more is 6 and 2 more makes 8.

Joyce: Fantastic, good strategy. Thank you for explaining that to me so clearly. Can someone else explain how they went on from there?

Sam: Once you know what the star is worth, you can find out the other symbols.

Joyce: Good thought. Can you give us an example of how you worked out one of the other symbols?

Sam: Well, it's hard to explain. In the bit where you have two stars and two spades, you can add the two stars together to get 4. As they all come to 16, you know that the spade must be worth 6.

Joyce: I'm not sure I followed what you said at the end. How did you know that the spade must be worth 5?

Sam: It's easy, miss. The two spades must be worth 12 all together, because the two stars are worth 4 and the total is 16. If the two spades are worth 10, then each one must be worth 6.

Joyce: Fantastic, now I've got it. Does anybody want to ask Sam about how he got 6?

Kim: Now it's easy. Once you know a couple of the symbols, you can find out the others.

Joyce: Can you give us an example of another symbol that you found out?

Kim: Easy. In the column with two stars, a spade and the dollar, you only have to find out the dollar thing. You know all the others.

This discussion is less about the different calculation strategies and more about the problem-solving strategies that the children are using. The children are still using and applying a number of different mental mathematics strategies.

## The vocabulary of reasoning

Marcus Witt (2014) also explores some key words associated with mathematical reasoning. Use of these words in discussions of mathematics can be a sign that reasoning is being used. This is useful when teaching (as children's use of these

## 1 Introduction

words is a sign of their learning) but can also be useful for you when developing your own understanding.

### Suggested key words and phrases to prompt reasoning

| Question (key words in italics)  | Answer (key words in italics)   |
|--|---|
| <i>How did you know</i> that the answer was 45?  | I knew <i>because</i> 20 and 20 is 40, <i>so</i> 20 and 25 must be 40 and 5 more.   |
| <i>Why do you think</i> the answer will be an even number?   | <i>I think</i> it will be even <i>because</i> an even number added to an even number <i>always</i> makes an even number.  |
| <i>What made you so sure</i> that the shape couldn't be a square?                                    | <i>If</i> the shape was a square, <i>then</i> it would have to have only right angles. As that corner isn't a right angle, the shape can't be a square.                                   |
| <i>Can you explain why</i> you added one back?   | <i>If</i> you are trying to take away 9, but you have actually taken away 10, <i>then</i> you've taken away too much, <i>so</i> you have to give one back.                                |
| <i>Tell us how you knew</i> it would be more than 10.  | <i>Whenever</i> you add two numbers together, unless one's a minus number, they <i>will always make</i> a bigger number. <i>So</i> 10 and something <i>will always be</i> bigger than 10. |
| <i>What do you think will happen</i> when we add 10 to this number? <i>Why?</i>                      | <i>I think that</i> the digit in the middle column will go up by one, but the others won't change, <i>because</i> we are only adding a 10, so only the tens column will change.           |
| <i>Is it always true</i> that adding 10 will only change the tens digit?                             | It's not always true, because sometimes, <i>if</i> the tens digit is already a 9, <i>then</i> the tens digit will go to zero and the hundreds digit will go up by one.                    |
| <i>Is it possible</i> for a fraction to have a bigger number on the top than the bottom? <i>How?</i> | <i>It is possible, because</i> sometimes a fraction is more than one, like 5 quarters, <i>so therefore</i> you would have to write the 5 first and then out of 4.                         |
| <i>What makes you think</i> these are a group?   | All those are a group <i>because</i> they all end in a 5, or a 0. <i>But</i> , actually, those others could be a different group <i>because</i> they are all less than 12.                |

The language of mathematical reasoning will become more familiar to you as you begin to consolidate your learning and start working with children. It is important to be aware of the importance of reasoning as you begin your training and to be mindful of the ways you can develop your reasoning skills.

## The Teachers' Standards

This book refers mostly to the mathematics-related subject standards you will be required to demonstrate in order to be awarded QTS. (See Mooney et al., 2018 for pedagogical and professional theory and practice.) This explicitly fulfils the following Standards:

A teacher must:

### 3. Demonstrate good subject and curriculum knowledge

- have a secure knowledge of the relevant subject(s) and curriculum areas, foster and maintain pupils' interest in the subject, and address misunderstandings

- demonstrate a critical understanding of developments in the subject and curriculum areas, and promote the value of scholarship

#### **4. Plan and teach well structured lessons**

- impart knowledge and develop understanding through effective use of lesson time
- promote a love of learning and children's intellectual curiosity
- contribute to the design and provision of an engaging curriculum within the relevant subject area(s)

#### **8. Fulfil wider professional responsibilities**

- take responsibility for improving teaching through appropriate professional development

In addition to these obvious standards, by having a secure subject knowledge you will also be able to support your work in meeting many of the other standards that involve giving appropriate feedback to children, planning for progression, etc.

This book covers the full range of the primary mathematics curriculum but also goes beyond to look at further aspects. The consideration given to mathematical language, and reasoning and proof should ensure you are able to make the connections between mathematical concepts and go some way towards gaining the profound understanding mentioned earlier.

## **Curriculum context**

We have focused in this book on the core areas of mathematics subject knowledge and understanding that you will need to develop the mathematical knowledge and understanding of the children you work with.

A closer look at the Purpose of Study of the National Curriculum for mathematics is useful here as an introduction to the kind of knowledge required to 'teach' children.

### **Purpose of study**

*Mathematics is a creative and highly interconnected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject. (DfE 2013b, page 3)*

Understanding the context of the curriculum helps you to make sense of how your children have been taught and why. Therefore, in each chapter we have included context about the primary National Curriculum programmes of study and the Early Learning Goals for children in the Early Years Foundation Stage in mathematics.

## The Early Years Foundation Stage

The Statutory Framework for the Early Years Foundation Stage (EYFS) sets the standards that all early years providers must meet to ensure that children learn and develop well and are kept healthy and safe. It promotes teaching and learning to ensure children's 'school readiness' and gives children the broad range of knowledge and skills that provide the right foundation for good future progress through school and life. Children will remain in the EYFS until the end of the academic year in which they turn five years of age. In practice this will mean that children will follow EYFS until they finish their Reception year.

There are seven areas of learning and development that shape the educational programmes in early years settings. The three *prime* areas are:

- communication and language;
- physical development; and
- personal, social and emotional development.

The four *specific* areas are:

- literacy;
- mathematics;
- understanding the world; and
- expressive arts and design.

For the purposes of this book we shall focus on these, specifically the area of learning and development in 'mathematics', which *involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measure* (DfE, 2017a, page 8). Crucially, the focus in mathematics is that children talk about and apply mathematics in a way that makes common sense to them (Tickell, 2011, page 103).

## Mathematics in the National Curriculum

The primary National Curriculum for mathematics is split into three key stages: Key Stage 1 (5–7 years), Lower Key Stage 2 (7–9 years) and Upper Key Stage 2 (9–11 years). Each key stage includes attainment targets, which state what pupils are expected to know, apply and understand of the matters, skills and processes specified in the mathematics programme of study. The mathematics National Curriculum in England has three aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language;
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Furthermore, the National Curriculum explains how:

*Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects. (DfE 2013b, page 3)*

In order to support children with this, you need a good subject knowledge to see the connections, spot patterns, be fluent, reason mathematically and solve problems yourself.

## Assessment of primary mathematics

In England, all schools can develop and deploy their own approaches to formative assessment to support pupil attainment and progression. Each school's assessment framework should be built into their school curriculum, so that

## 1 Introduction

schools can check what children have learned and whether they are on track to meet expectations at the end of the key stage, and so that they can report regularly to parents.

At the time of writing, the DfE have recently confirmed that the external testing for mathematics will continue at the end of Key Stage 2. In the Government's response (DfE, 2011b) to the Bew Review (2011) into Key Stage 2 testing, assessment and accountability they stated:

*It is important that mathematics tests are accessible to all pupils and do not unfairly disadvantage weaker readers. At the time of publication, The Standards and Testing Agency was reviewing all future National Curriculum Tests in mathematics to ensure that they remain accessible to all pupils, and that they are primarily tests of mathematics rather than reading. (DfE, 2011b: 10)*

In September 2017, the Department for Education published the *Primary Assessment in England Government Consultation Response*. This document outlines the government response to the consultation on the long-term future of primary assessment that ran from March 2017 to June 2017. This consultation

*sought views on a number of key proposals about the future of the statutory primary assessment system. These included assessment in the early years, the starting point for measuring the progress that pupils make at primary school, statutory end-of-key stage teacher assessment and proposals to ensure that we have a proportionate assessment system. (DfE, 2017b, page 4)*

The document gives details of the government's aim to

*remove the statutory requirement for schools to report teacher assessment judgements in English reading and mathematics at the end of key stage 2 from the 2018 to 2019 academic year onwards, once the relevant legislation has been amended. We believe that removing this duty to report judgements against the statutory teacher assessment frameworks will reduce burdens for teachers. (DfEb, 2017, pages 24–5)*

It makes clear, however, that schools' statutory requirement to report pupils' attainment and progress to parents will remain.

Also outlined is the government's plan to

*make assessments at the end of key stage 1 (both national curriculum tests and statutory teacher assessment) non-statutory as soon as the reception baseline assessment has become fully established. If possible, we intend to make this change from the 2022 to 2023 academic year onwards. (DfE, 2017b, page 20)*

Finally, the document explains clear plans to introduce the Multiplication Tables Check (a statutory test announced in 2016) at the end of Year 4.

## The Primary Framework for Literacy and Mathematics

While it was not statutory, the Primary Framework for Mathematics contained guidance and resources to help teachers to plan and teach mathematics to 3–11-year-old children. Many schools used the framework in their planning because by doing so they were meeting the requirements of the National Curriculum at the time.

There were seven strands in the Framework for Mathematics. These included:

- using and applying mathematics;
- counting and understanding number;
- knowing and using number facts;
- calculating;
- understanding shape;
- measuring;
- handling data.

The framework encouraged a planning and teaching style for Key Stages 1 and 2 that used ‘blocks’. These required teachers to consider links between different strands. This revised approach to teaching mathematics not only included the opportunity for teachers to be more creative in their approach to teaching, it also required them to have a secure subject knowledge.

Although the requirement to have regard for the primary national strategies has been discontinued, you may find that schools and individual teachers you work with are still using whole blocks or adapted units of this framework in their medium-term and long-term planning for mathematics, or other elements, such as the three-part numeracy lesson.

## Outcomes

This book can be used to start to address the development of your mathematical subject knowledge. Through developing subject knowledge of mathematics, including patterns and relationships, the ability to communicate, discuss, generalise, hypothesise and relate mathematics to the wider world, you will develop

positive and active attitudes towards and greater enthusiasm for mathematics. It should also stimulate a sense of curiosity and enjoyment and encourage communication of these positive attitudes to children, resulting in teaching that is effective, challenging and extremely rewarding.

By using this book to support your own subject knowledge development, you will be able to learn the knowledge and develop the understanding required to teach primary mathematics, and the appropriate elements of 'Problem solving, reasoning and numeracy' in the EYFS.

So that you can check on how well you have assimilated the subject knowledge and test your understanding, you may wish to try the self-assessment questions related to each aspect that we address. You will find these in a separate chapter towards the end of the book. The answers to these questions are provided for you in a separate section.

For those undertaking credits for a Master's Degree, we have included suggestions for further work and extended study at the end of each chapter in a section called 'M-Level Extension'.

## FURTHER READING

To support you in understanding the curriculum context, you may find it helpful to refer to some of the following documentation:

DfE (2013) *Mathematics Programmes of Study: Key Stages 1 and 2. National Curriculum in England*. London: DfE. Available at [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/239129/PRIMARY\\_National\\_Curriculum\\_-\\_Mathematics.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/239129/PRIMARY_National_Curriculum_-_Mathematics.pdf)

DfE (2017) *Statutory Framework for the Early Years Foundation Stage: Setting the Standards for Learning, Development and Care for Children from Birth to Five*. London: DfE. Available at <https://www.gov.uk/government/publications/early-years-foundation-stage-framework--2>

Ofsted (2007) *Mathematics: Understanding the Score – Improving Practice in Mathematics Teaching at Primary Level*. London: Ofsted.